14 problems were used to perform the comparisons, a summary of features for each test problem is presented in the following Table.

Table 1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Problem | number of variables | number of constraints | objective function | constraint |
| Ackley | 20 | 0 | nonlinear | - |
| Griewank | 20 | 0 | nonlinear | - |
| Rastrigin | 20 | 0 | nonlinear | - |
| Rosenbrock | 20 | 0 | quartic | - |
| Zakharov | 20 | 0 | quartic | - |
| G01 | 13 | 9 | quadratic | linear inequality |
| G02 | 20 | 2 | nonlinear | nonlinear inequality |
| G03 | 10 | 1 | polynomial | nonlinear equality |
| G04 | 5 | 6 | quadratic | nonlinear inequality |
| G05 | 4 | 5 | cubic | linear inequality |
| G06 | 2 | 2 | cubic | nonlinear inequality |
| G07 | 10 | 8 | quadratic | linear inequality  &&  nonlinear inequality |
| G08 | 2 | 2 | nonlinear | nonlinear inequality |
| G09 | 8 | 6 | polynomial | nonlinear inequality |

In order to evaluate the performance of two algorithms on the problems, each algorithm is individually executed 10 times using two different termination criteria, resulting in a total of 20 runs per algorithm per problem. The same random seed is used within each run, but it is changed for every new run.

The default parameters are utilized for both algorithms, with a population of 25 for Particle Swarm Optimization (PSO) and a population of 100 for Genetic Algorithm (GA). Two factors, namely quality and consistency, are considered when assessing the performance of the algorithms on each problem.

To measure the quality of the algorithms, the shortest execution time and shortest generational distance are employed. The execution time represents how quickly the algorithms find a solution, while the generational distance measures the closeness of the solutions to the optimal solution.

For evaluating the consistency of the algorithms, the average and standard deviation are calculated. These statistics provide insights into how stable and predictable the performance of the algorithms is across different runs.

The termination criterion based on the change in the objective space (ftol) is used to determine how long each algorithm takes to reduce the change in the objective space to a value less than 1e-6. For problems that the termination criterion cannot be reached, an upper bound of 100,000 for evaluations is defined to stop the process. Additionally, a maximum evaluation limit of 50000 is set to assess the ability of each algorithm to converge to the global optimum within a constrained number of evaluations.

Table 2

|  |  |  |  |
| --- | --- | --- | --- |
| Problem | | Execution time of PSO (seconds) | Execution time of GA (seconds) |
| Ackley | Best result | **0.8055** | 4.674 |
| Average | **3.516** | 6.518 |
| Std. dev. | **1.219** | 1.295 |
| Griewank | Best result | **0.6346** | 4.572 |
| Average | **1.432** | 6.851 |
| Std. dev. | **0.8687** | 1.586 |
| Rastrigin | Best result | **0.4212** | 4.856 |
| Average | **1.293** | 6.729 |
| Std. dev. | **1.352** | 1.837 |
| Rosenbrock | Best result | **0.3754** | 9.240 |
| Average | **1.460** | 24.48 |
| Std. dev. | **0.6753** | 4.326 |
| Zakharov | Best result | **0.1935** | 0.4206 |
| Average | **1.145** | 24.48 |
| Std. dev. | **0.7526** | 32.84 |
| G01 | Best result | **2.422** | 2.751 |
| Average | **1.872** | 2.003 |
| Std. dev. | **0.3510** | 0.6495 |
| G02 | Best result | **0.8118** | 2.888 |
| Average | **1.422** | 4.356 |
| Std. dev. | **0.6056** | 1.004 |
| G03 | Best result | **0.1056** | 0.3853 |
| Average | **0.2279** | 1.236 |
| Std. dev. | **0.07467** | 1.149 |
| G04 | Best result | **0.7595** | 1.550 |
| Average | **1.980** | 2.294 |
| Std. dev. | 0.8794 | **0.5610** |
| G05 | Best result | **0.7289** | 3.447 |
| Average | **1.189** | 7.195 |
| Std. dev. | **0.2382** | 3.4599 |
| G06 | Best result | **0.2901** | 0.3214 |
| Average | **0.8172** | 8.594 |
| Std. dev. | **0.5080** | 5.626 |
| G07 | Best result | **0.8015** | 2.613 |
| Average | **1.693** | 3.400 |
| Std. dev. | 0.7864 | **0.5000** |
| G08 | Best result | **0.1853** | 0.6235 |
| Average | **0.2744** | 0.9821 |
| Std. dev. | **0.1079** | 0.2004 |
| G09 | Best result | **0.6136** | 2.037 |
| Average | **1.115** | 3.007 |
| Std. dev. | **0.4153** | 0.7768 |

Table 2 shows the execution time each algorithm needed to reach a convergence point where the change in the objective space is less than 1e-6 in each problem. It is evident that, on average, PSO requires less execution time and achieves better results in each problem. Moreover, GA has greater standard deviation in most cases, suggesting that its convergence speed is less stable and consistent.

Table 3

|  |  |  |  |
| --- | --- | --- | --- |
| Problem | | PSO | GA |
| Ackley | Best result | **3.863e-11** | 0.02349 |
| Average | **0.002451** | 0.01570 |
| Std. dev. | 0.007352 | **0.005063** |
| Griewank | Best result | **0** | 0.004816 |
| Average | 0.2336 | **0.02779** |
| Std. dev. | 0.3323 | **0.01546** |
| Rastrigin | Best result | 0.9950 | **0.001405** |
| Average | 10.09 | **0.002813** |
| Std. dev. | 6.145 | **0.001195** |
| Rosenbrock | Best result | 9.433 | **0.1863** |
| Average | 16.06 | **14.80** |
| Std. dev. | **2.585** | 4.927 |
| Zakharov | Best result | **6.125e-6** | 0.4675 |
| Average | **0.004156** | 1.9345 |
| Std. dev. | **0.008932** | 1.046 |
| G01 | Best result | **0** | 0.0006182 |
| Average | 1.632 | **0.7185** |
| Std. dev. | 1.107 | **1.096** |
| G02 | Best result | 0.2585 | **0.01616** |
| Average | 0.3823 | **0.04618** |
| Std. dev. | 0.06860 | **0.02544** |
| G03 | Best result | **0.3317** | 0.4356 |
| Average | **0.6814** | 0.8311 |
| Std. dev. | 0.2234 | **0.1620** |
| G04 | Best result | **0** | 1.839 |
| Average | **1.176e-8** | 6.623 |
| Std. dev. | **3.366e-8** | 5.172 |
| G05 | Best result | - | - |
| Average | - | - |
| Std. dev. | - | - |
| G06 | Best result | **6.628e-6** | 11.14 |
| Average | **8.144e-5** | 68.12 |
| Std. dev. | **5.917e-5** | 45.12 |
| G07 | Best result | 0.1948 | **0.1324** |
| Average | **1.620** | 4.393 |
| Std. dev. | **1.106** | 4.689 |
| G08 | Best result | **0** | 2.032e-8 |
| Average | **0** | 8.091e-15 |
| Std. dev. | **0** | 5.018-8 |
| G09 | Best result | **0.03165** | 0.2025 |
| Average | **0.1397** | 0.6729 |
| Std. dev. | **0.0690** | 0.3611 |

Table 3 shows the generational distance of each algorithm for each problem, which is the distance from the obtained solutions to the Pareto-front. In terms of average generation distance, PSO outperforms GA in 8 problems and achieves better best results in 9 problems. Moreover, PSO has lower standard deviation in 7 problems. It shows PSO has better performance in terms of quality and consistency than GA. It is worth noting that the best results found by PSO in Griewank, G01, G04, and G08 are 0, meaning PSO was able to discover the pre-defined optimal solution. Additionally, neither PSO nor GA were able to find feasible solutions in G05.

Table 4

|  |  |  |  |
| --- | --- | --- | --- |
| Problem | | Time | GD |
| Ackley | Best result | 89.14 | 18.37 |
| Average | 91.97 | 19.13 |
| Std. dev. | 2.952 | 0.4091 |
| Griewank | Best result | 31.21 | 0.4040 |
| Average | 33.38 | 1.242 |
| Std. dev. | 1.255 | 0.8157 |
| Rastrigin | Best result | 40.99 | 0.5101 |
| Average | 50.04 | 2.746 |
| Std. dev. | 4.124 | 1.734 |
| Rosenbrock | Best result | 20.10 | 4.771e-05 |
| Average | 21.89 | 0.009471 |
| Std. dev. | 1.780 | 0.01108 |
| Zakharov | Best result | 21.69 | 3.858e-06 |
| Average | 23.98 | 0.1515 |
| Std. dev. | 1.606 | 0.4100 |
| G01 | Best result | **0** | 0.0006182 |
| Average | 1.632 | **0.7185** |
| Std. dev. | 1.107 | **1.096** |
| G02 | Best result | 0.2585 | **0.01616** |
| Average | 0.3823 | **0.04618** |
| Std. dev. | 0.06860 | **0.02544** |
| G03 | Best result | **0.3317** | 0.4356 |
| Average | **0.6814** | 0.8311 |
| Std. dev. | 0.2234 | **0.1620** |
| G04 | Best result | **0** | 1.839 |
| Average | **1.176e-8** | 6.623 |
| Std. dev. | **3.366e-8** | 5.172 |
| G05 | Best result | - | - |
| Average | - | - |
| Std. dev. | - | - |
| G06 | Best result | **6.628e-6** | 11.14 |
| Average | **8.144e-5** | 68.12 |
| Std. dev. | **5.917e-5** | 45.12 |
| G07 | Best result | 0.1948 | **0.1324** |
| Average | **1.620** | 4.393 |
| Std. dev. | **1.106** | 4.689 |
| G08 | Best result | **0** | 2.032e-8 |
| Average | **0** | 8.091e-15 |
| Std. dev. | **0** | 5.018-8 |
| G09 | Best result | **0.03165** | 0.2025 |
|  |  |  |
|  |  |  |